ISSN-2394-5125 VOL 07, ISSUE 10, 2020

HESITANT FUZZY MAXIMAL, MINIMAL OPEN AND CLOSED SETS

M.Sankari*

*Department of Mathematics, Lekshmipuram College of Arts and Science, Neyyoor, Kanyakumari, Tamilnadu -629802

Abstract
The aim of this article is to investigate few conditions for hesitant fuzzy disconnectedness via hesitant fuzzy maximal and minimal open sets; we show that if a hesitant fuzzy space having a hesitant fuzzy set which is hesitant fuzzy minimal and maximal then either this fuzzy set is the only nontrivial hesitant fuzzy open set in the hesitant fuzzy space or the hesitant fuzzy space is hesitant fuzzy disconnected.

Key words and phrases : Hesitant fuzzy minimal open; hesitant fuzzy maximal open; hesitant fuzzy clopen; hesitant fuzzy disconnected.

AMS(2000)subjectclassification:54A40,03E72.

1. Introduction

Zadeh[8] established fuzzy set in 1965 and Chang[1] introduced fuzzy topology in 1968. As an addendum to fuzzy sets, the notion hesitant fuzzy set introduced by Torra[7] in 2010. In 2019 Deepak et.al.[2] introduced hesitant fuzzy topological space and extended the study to hesitant connectedness and compactness in hesitant fuzzy topological space. After the study and effect of these notions, we develop in this article some properties and connection of hesitant fuzzy minimal and maximal open and closed sets.

2. Preliminaries

Definition 2.1.[6]*A proper nonzero hesitant fuzzy open set ξ of X is called a (i) hesitant fuzzy minimal open set if ξ and $h^0$are only hesitant fuzzy open sets contained in ξ .(ii) hesitant fuzzy maximal open set if $h^1$and ξ are only hesitant fuzzy open sets containing ξ*

Theorem 2.1.[6] If ξ is a hesitant fuzzy maximal open set and υ be a hesitant fuzzy open subset, then either $\xi \vee \upsilon = h^1$or $\upsilon < \xi$.

Theorem2.2.[6] If ξ is a hesitant fuzzy minimal open set and υ be a hesitant fuzzy open subset, then either $\xi \wedge \upsilon = h^0$ or $\xi < \upsilon$.

Definition2.2.[6] *A proper nonzero hesitant fuzzy closed set η of X is called a*

(i) hesitant fuzzy maximal closed set if any hesitant fuzzy closed set which contains η is h^1or η .

(ii) hesitant fuzzy minimal closed set if any hesitant fuzzy closed set which is contained in η is h^0or η.

Theorem 2.3.[6] If η is a hesitant fuzzy maximal closed set and ω is a hesitant fuzzy closed set, then either $\eta \vee \omega = h^1$ or $\omega < \eta$.

Theorem 2.4.[6]If η is a hesitant fuzzy minimal closed set and ω is any hesitant fuzzy closed set, then either $\eta \wedge \omega = h^0$ or $\eta < \omega$.

Definition 2.3.[4]A hesitant fuzzy set h in X is a function $h{:}X \rightarrow P[0,1]$, where $P[0,1]$ represents the power set of [0,1].
We define the hesitant fuzzy empty set h^0 (resp. whole set h^1) is a hesitant fuzzy set in X as follows: $h^0(x) = \varphi$ (resp.$h^1(x) = [0,1]$), $\forall x \in X$. $HS(X)$ stands for collection of hesitant fuzzy set in X.

Definition 2.4.[3]Two hesitant fuzzy set h_1, $h_2 \in HS(X)$ such that $h_1(x) \subset h_2(x), \forall x \in X$, then h_1 is contained in h_2.

Definition 2.5.[3] Two hesitant fuzzy set h_1 and h_2 of X are said to be equal if $h_1 \subset h_2$ and $h_2 \subset h_1$.

Definition 2.6.[4] Let $h \in HS(X)$ for any non empty set X. Then h^c is the complement of h which is hesitant fuzzy set in X such that $h^c(x) = [h(x)]^c = [0,1] \backslash h(x)$.

Definition 2.7.[5] Let (X, τ) be a hesitant fuzzy topological spaces. Let $x_\lambda \in H_p(X)$ and $N \in HS(\eta)$. Then the hesitant fuzzy neighbourhood N of x_λ is defined as iff or an hestitant fuzzy set $U \in \tau$ such that $x_\lambda \in U \subset N$.

Definition 2.8.[5] Let X be a nonempty set. A HFT τ of subsets X is said to be HFT on X if

(i) $h^0, h^1 \in \tau$.

(ii) $\bigcup_{i\in J} h_i \in \tau$ for each $(h_i)_{i\in J} \subset \tau$.

(iii) $h_1 \cap h_2 \in \tau$ for any $h_1, h_2 \in \tau$.

"The pair (X, τ) is called hesitant fuzzy topological spaces. The members of τ are called hesitant fuzzy open sets in X. A hesitant fuzzy set h in X is hesitant fuzzy closed set in (X, τ) if $h^c \in \tau$."

Definition 2.9. *[2]A hesitant fuzzy topological spaces X is said to be hesitant fuzzy disconnected if it has no proper hesitant fuzzy clopen set. A hesitant fuzzy set of X which is not hesitant fuzzy connected is called as hesitant fuzzy disconnected.*

3. Hesitant Fuzzy Maximal and Minimal Open Sets

Theorem 3.1. If ξ is a hesitant fuzzy maximal open set and υ is a hesitant fuzzy minimal open set in X, then either $\upsilon < \xi$ or the space is hesitant fuzzy disconnected.

Proof. Since the maximality condition of ξ by theorem 2.1, it could be either $\xi \vee \upsilon = h^1$ or $\upsilon < \xi$. Since the minimality condition of υ by theorem 2.2, we can have either $\xi \wedge \upsilon = h^0$ or $\upsilon < \xi$. When $\xi \vee \upsilon = h^1$, $\upsilon < \xi$ gives $\xi = h^1$; when $\xi \wedge \upsilon = h^0$, $\upsilon < \xi$ gives $\upsilon = h^0$. Therefore the probable occurrences are $\xi \vee \upsilon = h^1$, $\xi \wedge \upsilon = h^0$ and $\upsilon < \xi$. If $\xi \vee \upsilon = h^1$, $\xi \wedge \upsilon = h^0$, then the space is hesitant fuzzy disconnected.

Remark 3.2. $\xi \vee \upsilon = h^1$, $\xi \wedge \upsilon = h^0$, imply $\xi = h^1 - \upsilon$. In Theorem 3.1, if $\upsilon \not< \xi$, then ξ and υ are hesitant fuzzy closed. Now theorem 3.1could be rewritten in this way: if ξ is a hesitant fuzzy maximal open set and υ is a hesitant fuzzy minimal open set of a hesitant fuzzy topological spaces X, then either $\upsilon < \xi$ or $\xi = h^1 - \upsilon$.

Theorem 3.3. If η is a hesitant fuzzy topological spaces in X which is both hesitant fuzzy maximal open and hesitant fuzzy minimal open, then either this hesitant fuzzy set is the only nontrivial hesitant fuzzy open set in the hesitant fuzzy space or the hesitant fuzzy space is hesitant fuzzy disconnected.

Proof. Let ξ be both hesitant fuzzy maximal open and hesitant fuzzy minimal open, and υ be any hesitant fuzzy open set. Then we get $\xi < \xi \vee F$. By the maximality condition of ξ, it could be succeeding two cases:

Case I: As $\xi = \xi \vee \upsilon$, then $\upsilon < \xi$. Since ξ is hesitant fuzzy minimal openset, we have $\upsilon = h^0$ or $\upsilon = \xi$.

Case II: Assume $\xi \vee \upsilon = h^1$. Considering ξ as a hesitant fuzzy minimal openset, weget by Theorem 2.2, $\xi \wedge \upsilon = h^0$ or $\xi < \upsilon$. Since ξ is hesitant fuzzy maximal open , $\xi < \upsilon$ implies $\xi = \upsilon$ or $\upsilon = h^1$.

Considering all the cases, we get $\xi = \upsilon$ or $\xi \vee \upsilon = h^1$ and $\xi \wedge \upsilon = h^0$. If $\xi \vee \upsilon = h^1$ and $\xi \wedge \upsilon = h^0$, then the space is hesitant fuzzy disconnected.

Remark 3.4. The following facts to be taken care of dealing hesitant fuzzy minimal open and hesitant fuzzy maximal open in a hesitant fuzzy topological spaces:

(i) If there is only one proper hesitant fuzzy open set in X, then this hesitant fuzzy set act as a hesitant fuzzy maximal open and hesitant fuzzy minimal open.

(ii) If there are only two proper disjoint hesitant fuzzy open sets in X, then both act as hesitant fuzzy maximal open and hesitant fuzzy minimal open.

(iii) If ξ and υ are only two proper hesitant fuzzy open sets in X such that $\xi < \upsilon$, then ξ is a hesitant fuzzy minimal open set and υ is a hesitant fuzzy maximal open set in the hesitant fuzzy space.

But there could not exist a hesitant fuzzy set which is both hesitant fuzzy maximal open and hesitant fuzzy minimal open in a hesitant fuzzy disconnected space (see Example 3.7).The above is true for choice of taking hesitant fuzzy minimal closed and hesitant fuzzy maximal closed in X .

Corollary 3.5. If η is a hesitant fuzzy closed set and ξ is both hesitant fuzzy maximal open and hesitant fuzzy minimal open in X, then either $\xi = h^1 - \eta$ or $\xi = \eta$.

Proof. Given η is a hesitant fuzzy closed set and ξ is both hesitant fuzzy maximal open and hesitant fuzzy minimal open. So $h^1 - \eta$ is a hesitant fuzzy open set. In such a way that proof of theorem 3.3, weget $\xi = h^1 - \eta$ or $\xi \vee (h^1 - \eta) = h^1$ and $\xi \wedge (h^1 - \eta) = h^0$. Both $\xi \vee (h^1 - \eta) = h^1$and $\xi \wedge (h^1 - \eta) = h^0$ imply $\xi = \eta$.

Corollary 3.6. If ξ is both hesitant fuzzy maximal open and hesitant fuzzy minimal open in X, then either ξ is the only proper hesitant fuzzy open set or ξ and $h^1 - \xi$ are the only proper hesitant fuzzy open sets in hesitant fuzzy space.

Proof. Let υ be any proper hesitant fuzzy open set of the hesitant fuzzy space. In such a way that proof of Theorem 3.3, weget $\xi = \upsilon$ or $\xi \vee \upsilon = h^1$ and $\xi \wedge \upsilon = h^0$. Both $\xi \vee \upsilon = h^1$ and $\xi \wedge \upsilon = h^0$ imply $\upsilon = h^1 - \xi$.

Example3.7. Let $X = \{a, b, c\}$and $\tau = \{h^0, \xi_1, \xi_2, \xi_3, \xi_4, h^1\}$where

$\xi_1(a) = \{0.2\}$, $\xi_2(b) = \{0.3\}$, $\xi_3(c) = \{0.4\}$

$\xi_2(a) = [0,0.2)$, $\xi_2(b) = [0,0.3)$, $\xi_2(c) = [0,0.4)$

$\xi_3(a) = (0,0.2]$, $\xi_3(b) = (0,0.3]$, $\xi_3(c) = (0,0.4]$and

$\xi_4(a) = [0.2,1]$, $\xi_4(b) = [0.3,1]$, $\xi_4(c) = [0.4,1]$.

Hence (X, X) is hesitant fuzzy topological space. The space (X, τ) is hesitant fuzzy disconnected with a separation ξ_2 and ξ_4. But the space has no hesitant fuzzy open set which is both hesitant fuzzy maximal open and hesitant fuzzy minimal open.

Theorem 3.8. If ω_1 and ω_2 are two different hesitant fuzzy maximal open sets in X with $\omega_1 \wedge \omega_2$ is a hesitant fuzzy closed set, then X is hesitant fuzzy disconnected.

Proof. Since ω_1 and ω_2 are hesitant fuzzy maximal open, we have $\omega_1 \vee \omega_2 = h^1$. We put $\xi = \omega_1 - \omega_1 \wedge \omega_2$, $\upsilon = \omega_2$ or $\xi = \omega_1$, υ

$= \omega_2 - \omega_1 \wedge \omega_2$. We note that ξ, υ are disjoint hesitant fuzzy open sets with $\xi \vee \upsilon = h^1$. Therefore *X* is hesitant fuzzy disconnected.

Theorem 3.9. If κ is a hesitant fuzzy maximal open set, then either $Cl(\kappa)=h^1$ or $Cl(\kappa)=\kappa$.

Theorem 3.10. If there exists a hesitant fuzzy maximal open set which is not hesitant fuzzy dense in *X* , then the space is hesitant fuzzy disconnected.

Proof. Consider a hesitant fuzzy maximal open set κ which is not hesitant fuzzy dense in *X*. As Theorem 3.9, $\kappa = Cl(\kappa)$ could be written $\xi = \kappa$ and $\upsilon = h^1 - Cl(\kappa)$. Therefore (ξ, υ)is a hesitant fuzzy separation for *X* .

4. Hesitant Fuzzy Maximal and Fuzzy Minimal Closed Sets

Theorem 4.1. If η is a hesitant fuzzy set in *X* which is both hesitant fuzzy minimal closed and hesitant fuzzy maximal closed set, then either of the succeeding is true:

(i) η is the only proper hesitant fuzzy closed set in the hesitant fuzzy space.

(ii) If ω is a proper hesitant fuzzy closed set, then $\eta \vee \omega = h^1$ and $\eta \wedge \omega = h^0$.

Corollary 4.2. If υ is a hesitant fuzzy open set and η is both hesitant fuzzy maximal closed and hesitant fuzzy minimal closed set in *X*, then either $\eta = h^1 - \upsilon$ or $\eta = \upsilon$.

Corollary 4.3. If η is both hesitant fuzzy maximal closed and hesitant fuzzy minimal closed in *X*, then either η is the only proper hesitant fuzzy closed set or η and $h^1 - \eta$ are the only hesitant fuzzy closed sets in the hesitant fuzzy space.

Theorem4.4. If υ is a hesitant fuzzy closed and ξ is both hesitant fuzzy maximal open and hesitant fuzzy minimal closed, then either of the succeeding is true:

(i) $\upsilon < \xi < \eta$.

(ii) $\upsilon < \xi$ and $\xi \wedge \eta = h^0$.

(iii) $\xi \vee \eta = h^1$ and $\xi < \eta$.

(iv) $\xi \vee \eta = h^1$, $\xi \wedge \eta = h^0$.

Proof. If ξ as a hesitant fuzzy maximal open set by thorem 2.1,then $\upsilon < \xi$ or $\xi \vee \upsilon = h^1$. If ξ as a hesitant fuzzy minimal closed set by thorem 2.4, then $\xi < \eta$ or $\xi \wedge \eta = h^0$. Now $\upsilon < \xi$ and $\xi < \eta$ imply $\upsilon < \xi < \eta$. Remaining choices are $\upsilon < \xi$, $\xi \wedge \upsilon = h^0$; $\xi \vee \upsilon = h^1$, $\xi < \eta$ and $\xi \vee \upsilon = h^1$, $\xi \wedge \upsilon = h^0$.

Corollary4.5. If ξ is both hesitant fuzzy maximal open and hesitant fuzzy minimal closed, then ξ and $h^1 - \xi$ are only proper hesitant fuzzy clopen sets in the fuzzy space.

Proof. Let η be hesitant fuzzy clopen in h^1. Putting $\upsilon = \eta$ in Theorem 4.4, we get $\xi = \eta$ or $\xi = h^1 - \eta$.

Theorem4.6. If ξ is hesitant fuzzy maximal open as well as hesitant fuzzy maximal closed and η is hesitant fuzzy clopen, then either $\eta < \xi$ or $\xi \vee \eta = h^1$.

Proof. Analogous to Theorem 4.4.

Theorem 4.7. If υ is a hesitant fuzzy open and ξ is both hesitant fuzzy minimal open and hesitant fuzzy maximal closed, η is hesitant fuzzy closed, then either of the following is true:

(i) $\eta < \xi < \upsilon$.

(ii) $\xi < \upsilon$ and $\xi \vee \eta = h^1$.

(iii) $\xi \wedge \upsilon = h^0$ and $\eta < \xi$.

(iv) $\Xi \vee \eta = h^1$, $\xi \wedge \eta = h^0$

Corollary 4.8. If ξ is both hesitant fuzzy minimal open and hesitant fuzzy maximal closed, then ξ and $h^1 - \xi$ are only proper hesitant fuzzy clopen sets in the space.

Theorem4.9. Let δ, ξ be hesitant fuzzy open sets in *X* such that $\delta \wedge \xi \neq h^0$, δ. If ξ is a hesitant fuzzy minimal open set in (*X*, τ), then $\delta \wedge \xi$ is a hesitant fuzzy minimal open set in (δ, τ_δ)

Proof. If $\delta \wedge \xi$ is not a hesitant fuzzy minimal open set in (δ, τ_δ), then a hesitant fuzzy open set $\kappa \neq h^0$ in (δ, τ_δ) such that $\kappa \lneqq \delta \wedge \xi$. Since ξ is a hesitant fuzzy minimal open set in *X* and $\delta \wedge \xi \neq h^0$, we could get by theorem 2.2,$\xi < \delta$ gives $\delta \wedge \xi = \xi$. As δ being hesitant fuzzy open in *X*, κ is too. Hence we obtain a hesitant fuzzy set κ which is hesitant fuzzy open in *X* such that $h^0 \neq \kappa \lneqq \xi$ which contradicts that ξ is a hesitant fuzzy minimal open set in *X* .

References

[1] C.L.Chang, Fuzzy topological spaces, J. Math. Anal. Appl., 24(1968), 182-190.
[2] D. Deepak,B. Mathew,S.Mohn and H.A. Garg, Topological structure involving hesitant fuzzy sets. M. Intell. Fuzzy Syst. 2019,36, 6401-6412.
[3] D.Divakaran and S.J.John, Hesitant fuzzy rough sets through hesitant fuzzy relations, Ann. Fuzzy Math. Inform., 2014,8, 33-46.
[4] J.Kim, Y.B.Jun, P.K.Lim, J.G.Lee and K.Hur, The category of hesitant H-fuzzy sets. Ann. Fuzzy Math. Inform.2019,18, 57-74.
[5] J.G.Lee and K.Hur, Hesitant fuzzy topological spaces, Mathematics, 2020,8,188.

ISSN-2394-5125 VOL 07, ISSUE 10, 2020

[6] A. Swaminathan and S.Sivaraja, Hesitant fuzzy minimal and maximal open sets, J. Appl. and Pure Math. Vol. 5(2023), No.1-2, pp. 121-128.
[7] V.Torra, Hesitant fuzzysets. Int. J. Intel. Sys. 2010,25,529-539.
[8] L.A.Zadeh, Fuzzy sets, Information and control 8 (1965), 338-353.